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D-brane probes on G_2 orbifolds

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Abstract: We consider type-IIB string theory on a seven dimensional orbifold with holonomy in G_2. The motivation is to use D1-branes as probes of the geometry. The low energy theory on the D1-brane is a sigma-model with two real supercharges ($\mathcal{N} = (1,1)$ in two dimensional language). We study in detail the closed and open string sectors and propose a coupling of the twisted fields to the brane that modifies the vacuum moduli space so that the singularity at the origin is removed. Instead of coming from D-terms, which are not present here, the modification comes from a "twisted" mass term for the seven scalar multiplets on the brane. The proposed mechanism involves a generalization of the moment map.

Keywords: Bosonic Strings, D-branes.

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Contents

1. Introduction

Recently, there has been a focus of attention on manifolds with G_2 holonomy [1]. The physical motivation, (see, e.g. [2]–[5]) is very clear and very strong – M-theory on such manifolds gives rise to $\mathcal{N} = 1$ supersymmetry in $3 + 1$ dimensions. Phenomenologically interesting situations correspond to regions in the moduli space where the G_2 manifold develops singularities and non-perturbative effects arise. Investigations of the local physics near such singularities has already unearthed many new interesting phenomena. For a partial list of G_2-related literature, see [6]–[37].

In this paper, we will consider type-II string theory on orbifolds [38, 39, 40] whose holonomy is in G_2. Our motivation is to provide a setup where it is possible to apply the brane-probe techniques initiated by Douglas and Moore [41] and expanded in [42]–[45]. The idea is that the use of non-perturbative objects as probes unravels aspects of the geometry that depart from the classical picture. The effective spacetime geometry as 'seen' by the D-brane probes is the moduli space of vacua of the low-energy theory on their world-volume. The process by which D-branes resolve orbifold singularities is by the couplings of their world-volume theory to twisted bulk fields. These additional bulk couplings smooth-out the vacuum moduli space. In [41] this was applied to $\mathbb{C}^2/\mathbb{Z}_n$ orbifold singularities and the hyperKähler quotient construction [46, 47, 48] (see also [49]) was reproduced from a 'brany' point of view.

The fact that the amount of unbroken supersymmetry in our case is 1/4 of that in [41], makes our setup rather different. From the technical point of view, the low number of supercharges does not allow for D-terms — which in [41] were responsible for the resolution of the singularity! Instead of the D-terms, which are not present here, we propose that the singularity gets removed due to a "twisted" mass term for the seven scalar multiplets on the brane. Our analysis is valid for lengths r in the regime $l_P \ll r \ll \sqrt{\alpha'}$.

This paper is organized as follows: In section 2 we introduce our G_2 orbifold and in section 3 we discuss the closed string spectrum for type-II string theory on it, paying particular attention to the massless modes and the partition function. In section 4 we discuss the spectrum of open strings ending on D-branes and the corresponding gauge theory arising from their interactions. In section 5 we combine the results and propose a coupling of the twisted fields to the brane that removes the singularity at the origin. As a byproduct of the construction we propose a generalization of the moment map. We discuss the significance and limitations of our results in the last section 6.

2. The orbifold

To our knowledge, no classification of the possible discrete subgroups of G_2 is known. In this paper we will be concerned with what is arguably the simplest such orbifold: $\Gamma = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ where the three generators α, β and γ act on the last seven coordinates $X^3, \ldots X^9$ as follows:

	X^3	X^4	X^5	X^6	X^7	X^8	X^9
$\alpha :$	$-X^3$	$-X^4$	$-X^5$	$-X^6$	X^7	X^8	X^9
$\beta :$	$-X^3$	$-X^4$	X^5	X^6	$-X^7$	$-X^8$	X^9
$\gamma :$	$-X^3$	X^4	$-X^5$	X^6	$-X^7$	X^8	$-X^9$

Since we are interested only in the local physics near the singularity, we take our orbifold to be non-compact, i.e. of the form $\mathbb{R}^7/\Gamma$. By doing this we are implicitly assuming that we are focusing our attention to an open neighborhood U of a singular point in a *compact* manifold M, such that U is isometric to $\mathbb{R}^7/\Gamma$.

An important question, which we are not able to answer here, is whether M can be taken to be a G_2 manifold. There are several known compact G_2 manifolds, constructed by Joyce in [50], (For an occurrence of such orbifolds in the physics literature see [2]) by desingularizing compact orbifolds of the type $T^7/(\mathbb{Z}_2)^3$. The set of singular points in Joyce's examples consists of disjoint unions of singularities of the type $T^3 \times \mathbb{C}^2/\mathbb{Z}_2$, and are thus pretty well behaved. In particular, each singular patch can be desingularized much in the same way $\mathbb{C}^2/\mathbb{Z}_2$ can be desingularized to give the Eguchi-Hanson space [51]. Compared to the examples given in [50], our orbifold differs by the fact that none of the seven coordinates is compact and there is no associated "shift" in those coordinates. The singularity of our non-compact orbifold is a 'bad' one comparatively, in the sense that it cannot be smoothed-out by the method of [50].

It is easy to see that $\Gamma \subset G_2$ but $\Gamma \not\subset \mathrm{SU}(3)$. Recall that geometrically G_2 can be thought of as being generated by simultaneous rotations in two orthogonal planes in $\mathbb{R}^7$. (This is actually one way to show that G_2 is embedded in Spin(7)). Here we are dealing only with rotations by π, so we see that a group generated by seven-dimensional diagonal matrices with entries equal to ± 1 on the diagonal, will be in G_2 iff all the elements other than the identity have exactly four negative entries (thus rotating two orthogonal planes by π). This property is indeed satisfied by the group generated by the Spin(7) matrices

$$R(\alpha) = \mathrm{diag}(-1, -1, -1, -1, 1, 1, 1)$$

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$$
\begin{aligned}
R(\beta) &= \mathrm{diag}(-1,-1,1,1,-1,-1,1) \\
R(\gamma) &= \mathrm{diag}(-1,1,-1,1,-1,1,-1)\,.
\end{aligned} \tag{2.1}
$$

However, trying to add a generator of type $\mathrm{diag}(-1,-1,-1,1,-1,1,1)$ would generate matrices with only two negative entries, thus taking us outside of G_2 to the full Spin(7). To check that the orbifold group Γ is not a discrete subgroup of a smaller Lie subgroup of Spin(7) (for instance SU(3)) we must check that it acts non trivially on all the coordinates. For instance, the subgroup generated by $R(\alpha)$ and $R(\beta)$ alone is in SU(3).

3. The closed string spectrum

We begin by looking at the closed string spectrum (no branes) of type-II string theory on $\mathbb{R}^7/\Gamma$. We write the closed string mode expansion as $X^M(\tau,\sigma) = X^M_R(\tau-\sigma) + X^M_L(\tau+\sigma)$ where, as usual,

$$
\begin{aligned}
X^M_R(\tau-\sigma) &= \frac{1}{2}q^M + \sqrt{2\alpha'}(\tau-\sigma)\alpha^M_0 + i\sqrt{\frac{\alpha'}{2}}\sum_t \frac{\alpha^M_t}{t}e^{-2it(\tau-\sigma)} \\
X^M_L(\tau+\sigma) &= \frac{1}{2}q^M + \sqrt{2\alpha'}(\tau+\sigma)\tilde{\alpha}^M_0 + i\sqrt{\frac{\alpha'}{2}}\sum_t \frac{\tilde{\alpha}^M_t}{t}e^{-2it(\tau+\sigma)}\,.
\end{aligned} \tag{3.1}
$$

For $\mu = 0,1,2$ we have the usual closed string boundary conditions $X^\mu(\tau,0) = X^\mu(\tau,\pi)$ satisfied for $\alpha^\mu_0 = \tilde{\alpha}^\mu_0 = \sqrt{\frac{\alpha'}{2}}p^\mu$ and $t \in \mathbb{Z}-\{0\}$. The other boundary condition, (needed for i taking the appropriate values within $\{3,\cdots,9\}$ in each sector), is $X^i(\tau,0) = -X^i(\tau,\pi)$ and yields $\alpha^i_0 = \tilde{\alpha}^i_0 = 0$, $q^i = 0$ and $t \in \mathbb{Z}+1/2$.

The zero point energy in *all* twisted sectors is zero because there are always four half-odd moded bosonic coordinates. Let us consider type IIA for definiteness. Of the 64 untwisted massless d.o.f. in each of the four GSO sectors of the superstring ((NS_+,NS_+), (NS_+,R_-), (R_+,NS_+), and (R_+,R_-)) only 8 for each sector survive the orbifold projection. One can see this by looking at the field description of such d.o.f. — for instance, in the (NS_+,NS_+) sector we are left with $g_{\mu\nu}$ and $B_{\mu\nu}$, carrying no d.o.f. in $d = 2+1$ dimensions, the dilaton Φ and seven more scalars from g_{ii}, $i = 3,\ldots,9$ for a total of eight d.o.f. Similarly, in the (R_+,R_-) sector we have one d.o.f. from A_μ, dual to a scalar in $d = 2+1$ dimensions, and seven more scalars from A_{ijk} with the appropriate choice of indices. In type IIB the (R_+,R_-) sector is replaced by (R_+,R_+) and includes one scalar χ (the axion) and seven 3d scalars C^+_{ijkl}.

We can readily see that one-eighth of supersymmetry ($\mathcal{N} = 2$ in (2+1)-dimensions) is preserved by the orbifold by noting that only 8 of the 64 fermionic states survive. Alternatively, it is straightforward to check directly that there is exactly one spinor of Spin(7) invariant under the orbifold action.

One can obtain the same result by considering the explicit form of the action of α, β and γ on the states. For instance, in the (R_+,R_-) sector one needs the action $S(\alpha)$ $S(\beta)$ and $S(\gamma)$ of the generators on the fermionic zero modes.[1] Since all three generators

[1] We denote by S the eight dimensional representation of Γ acting on spinors, not to be confused with the seven dimensional one R, introduced before, which acts on vectors.

represent a rotation by π in two separate planes, it follows that, for instance,

$$S(\alpha) = \exp i\pi(\Sigma_{34} + \Sigma_{56}) \otimes \exp i\pi(\tilde{\Sigma}_{34} + \tilde{\Sigma}_{56}) = \Gamma^3\Gamma^4\Gamma^5\Gamma^6 \otimes \tilde{\Gamma}^3\tilde{\Gamma}^4\tilde{\Gamma}^5\tilde{\Gamma}^6 \tag{3.2}$$

when acting on the vacuum state in $|R_+\rangle \otimes |R_-\rangle$. Similarly

$$\begin{aligned} S(\beta) &= \Gamma^3\Gamma^4\Gamma^7\Gamma^8 \otimes \tilde{\Gamma}^3\tilde{\Gamma}^4\tilde{\Gamma}^7\tilde{\Gamma}^8 \\ S(\gamma) &= \Gamma^3\Gamma^5\Gamma^7\Gamma^9 \otimes \tilde{\Gamma}^3\tilde{\Gamma}^5\tilde{\Gamma}^7\tilde{\Gamma}^9 \,. \end{aligned} \tag{3.3}$$

The above matrices commute and thus can be simultaneously diagonalized leaving invariant 8 out of the 64 d.o.f. in the untwisted (R_+, R_-) sector.

As far as the twisted sectors are concerned, for each one of them and for each of the GSO sectors, we have exactly one degree of freedom. Thus we have a total of 7 twisted (NS_+, NS_+) scalar fields in (2+1)-dimensions (there are 7 twisted sectors), all corresponding to B-field moduli.[2]

One quick way to understand the above counting is to notice that each generator taken alone would give us a model identical to the $\mathbb{C}^2/\mathbb{Z}_2$ orbifold, for which it is well known that there are four d.o.f. in each twisted GSO sector. Keeping only those states that are invariant under the action of the other generators reduces their number by a quarter. Once again, this can be seen by looking at the action of the generators on the fermionic zero modes. Here, due to the anti-periodicity of some bosonic coordinates, there will be fermionic zero modes even in the (NS_+, NS_+) sector. For instance, in the α sector, we have

$$\begin{aligned} S(\alpha) &= \Gamma^3\Gamma^4\Gamma^5\Gamma^6 \otimes \tilde{\Gamma}^3\tilde{\Gamma}^4\tilde{\Gamma}^5\tilde{\Gamma}^6 \\ S(\beta) &= \Gamma^3\Gamma^4 \otimes \tilde{\Gamma}^3\tilde{\Gamma}^4 \\ S(\gamma) &= \Gamma^3\Gamma^5 \otimes \tilde{\Gamma}^3\tilde{\Gamma}^5 \,. \end{aligned} \tag{3.4}$$

These generators commute with each other and reduce the number of d.o.f. by a quarter.

This analysis carries over to type IIB virtually unchanged. The result is again that there is exactly one d.o.f. for each twisted sector in each one of the GSO sectors.

It is also instructive to look at the partition function of the theory. Let us denote by ${}_{g}\underset{h}{\square}$ the $8 \times 8 = 64$ sectors of the orbifold. All the sectors are of course zero due to the cancellation of bosons against the fermions but many of them are "*trivially zero*" in the sense that the bosonic and fermionic parts cancel separately. It is fairly easy to check that the only sectors that are *not* trivially zero are: ${}_{e}\underset{e}{\square}$, ${}_{e}\underset{h}{\square}$, and ${}_{h}\underset{h}{\square}$ for $h \in \{\alpha, \beta, \gamma, \alpha\beta, \alpha\gamma, \beta\gamma, \alpha\beta\gamma\}$, the 7 twisted sectors. Moreover, all ${}_{e}\underset{h}{\square}$ contributions are obviously the same and similarly for the ${}_{h}\underset{h}{\square}$.

The full partition function is thus

$$Z_{\mathbb{R}^7/\Gamma} = \frac{1}{8}\left({}_{e}\underset{e}{\square} + \sum_{h \neq e} {}_{e}\underset{h}{\square} + \sum_{h \neq e} {}_{h}\underset{h}{\square} \right) \equiv \frac{1}{8}\left({}_{e}\underset{e}{\square} + 7\, {}_{e}\underset{\alpha}{\square} + 7\, {}_{\alpha}\underset{\alpha}{\square} \right). \tag{3.5}$$

[2] In the previous version of the paper, we had incorrectly identified these scalars with metric deformations. We thank B. Acharya for pointing this out to us.

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We see immediately that only one eighth of the untwisted fields survives. As far as the twisted fields go, recall that orbifolding only by, say, α, we obtain

$$Z_{\mathbb{C}^2/\mathbb{Z}^2} = \frac{1}{2}\left(e\,\underset{e}{\square} + e\,\underset{\alpha}{\square} + \alpha\,\underset{\alpha}{\square} \right) . \tag{3.6}$$

We see that we have $2 \times (7/8)$ as many d.o.f. as in the $\mathbb{C}^2/\mathbb{Z}^2$ twisted sector, i.e. $2 \times (7/8) \times 4 = 7$.

Let us try to understand the massless spectrum from a geometrical perspective. Let us denote by $\tilde{\omega}^I$, $I = 1, \ldots, 7$, the two-forms on which we expand the B-field. In addition we have 7 (non-normalizable) $\Gamma-$invariant three-forms (and their dual four-forms) which we denote by ω^a, $a = 1, \ldots, 7$. These can be taken to be $dx^i \wedge dx^j \wedge dx^k$ with

$$(ijk) = \{(394),\ (358),\ (367),\ (475),\ (468),\ (569),\ (789)\} . \tag{3.7}$$

If our orbifold is though of as part of a compact singular manifold, the ω^a's will become normalizable.

The untwisted fields in the NS-NS sector are the dilaton Φ and 7 scalars coming from the $\Gamma-$invariant metric deformations θ_{ii}. In the R-R untwisted sector we have the axion χ and 7 one-forms c^a_μ obtained by reducing C^+_{MNKL} along ω^a. In the light-cone it is easy to see that the one-forms are dual to 7 scalars c^a which are obtained by reducing C^+_{MNKL} along ${}^*\omega^a$ — the Hodge star acting on the internal 7-dimensional space. In the twisted NS-NS sectors there are 7 scalars ϕ^I $I = 1 \cdots 7$, coming from the twisted B-field. Finally, the R-R twisted sectors contain 7 scalars c^I which come from expanding C_{MN} along $\tilde{\omega}^I$. All this is summarized in the following table

	NS-NS	R-R
untwisted :	Φ, θ_{ii}	χ, c^a
twisted :	ϕ^I	c^I

In the case of K3 surfaces, it is well known (see [53] for a nice review) that sixteen $\mathbb{C}^2/\mathbb{Z}_2$ blown-up orbifold singularities may be patched together to form the Kummer surface — a special kind of K3 surface. Moreover, Joyce's orbifolds mentioned in section 2 are just G_2 analogues of the Kummer surface. The reader may wonder whether our seven-dimensional orbifold is to any of these '7-dimensional Kummer surfaces' what $\mathbb{C}^2/\mathbb{Z}_2$ is to K3. However it is not. As already mentioned, our singularity is more complicated than the singularities in Joyce's examples and cannot be smoothed-out in the same way.

4. The open string spectrum

In order to perform the probe analysis of the next section we need the massless sector of the spectrum of D1 excitations, where the D1-brane is placed along $\mu = 0, 1$. The motion of the brane transverse to the orbifold is parameterized by the field X^μ, $\mu = 2$, while $\{X^i, i = 3, \ldots, 9\}$ parameterize the motion of D1 along the orbifolded directions. On the brane lives also a gauge field $A_\mu, \mu = 0, 1$ and, of course, the fermionic degrees of freedom required by supersymmetry.

The low-energy effective theory on the D1 brane world-volume is a linear 2d supersymmetric $\sigma-$model. Its supersymmetry can be determined in the following way. Let $Q_{L,R}$ be the supercharges associated to left, right-moving worldsheet degrees of freedom of type-IIB string theory. The closed-string sector is invariant under supersymmetry transformations of the form $\epsilon_L Q_L + \epsilon_R Q_R$ where $\epsilon_{L,R}$ are Majorana-Weyl 10d spinors, i.e. they are in the $\mathbf{16}_+$ of Spin(1, 9). A D1 brane along the $\mu = 0, 1$ directions is invariant under the subset of the above supersymmetry transformations which obey in addition $\epsilon_L = \Gamma^{01}\epsilon_R$. This means that ϵ_L (say) can be expressed in terms of ϵ_R leaving the latter as the only independent supersymmetry parameter. In other words the D1 breaks half the supersymmetry. It is useful to decompose ϵ_R under $\mathrm{Spin}(1,1)\otimes\mathrm{Spin}(8)\subset\mathrm{Spin}(1,9)$ so that $\epsilon_R \sim \mathbf{8}_{1/2}\oplus\mathbf{8}_{-1/2}$, where the subscripts denote Spin(1, 1) weight. Upon further compactifying 7 of the 8 transverse directions on a G_2 manifold (orbifold) only the singlets of G_2 survive as unbroken supersymmetry parameters. Noting that the $\mathbf{8}$ of Spin(8) decomposes under G_2 into $\mathbf{7}\oplus\mathbf{1}$, we conclude that the unbroken supersymmetry parameter transforms as the $\epsilon_R \sim \mathbf{1}_{1/2}\oplus\mathbf{1}_{-1/2}$ of $\mathrm{Spin}(1,1)\otimes G_2$. In other words the theory on the D1 brane will be a linear $(1, 1)$ $\sigma-$model.

Consistency conditions. In order to find the massless open string spectrum we follow a well known procedure (see, e.g. [54, 55]). Let ρ be the regular representation[3] of Γ. It is of course eight-dimensional and it is straightforward to see that for an appropriate choice of basis it takes the form

$$\rho(\alpha) = \mathbb{1}\otimes\mathbb{1}\otimes\sigma^3\,; \qquad \rho(\beta) = \mathbb{1}\otimes\sigma^3\otimes\mathbb{1}\,; \qquad \rho(\gamma) = \sigma^3\otimes\mathbb{1}\otimes\mathbb{1}\,. \tag{4.1}$$

The field X^μ, $\mu = 2$, satisfies

$$\rho(\alpha)X^2\rho(\alpha) = \rho(\beta)X^2\rho(\beta) = \rho(\gamma)X^2\rho(\gamma) = X^2\,, \qquad (X^2)^\dagger = X^2\,. \tag{4.2}$$

The solution of the above equations is

$$X^2 = \begin{pmatrix} x_1^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x_2^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x_3^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_4^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x_5^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & x_6^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_7^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_8^2 \end{pmatrix}, \tag{4.3}$$

where $x_1^2, \ldots, x_8^2 \in \mathbb{R}$.

The gauge field satisfies the same condition and is also given by a real, diagonal 8×8 matrix, implying a $\mathrm{U}(1)^8$ gauge symmetry. The diagonal U(1), which describes the center-of-mass of the branes, decouples and will play no role in the following. The 'real' gauge group is therefore $\mathrm{U}(1)^7$.

[3] Yet another representation, not to be confused with the previous ones R and S.

The fields $X^{3,\dots,9}$ satisfy

$$
\begin{aligned}
\rho(\alpha)X^i\rho(\alpha) &= R(\alpha)^i_j X^j\,,\\
\rho(\beta)X^i\rho(\beta) &= R(\beta)^i_j X^j\,,\\
\rho(\gamma)X^i\rho(\gamma) &= R(\gamma)^i_j X^j\,,\\
(X^i)^\dagger = X^i\,, \qquad & i = 3\dots 9\,.
\end{aligned}
\tag{4.4}
$$

The solution of the above equations can be given in terms of 28 complex numbers x^i, y^i, z^i, $w^i, i = 3, \dots, 9$ and reads

$$
X^3 = \begin{pmatrix}
0&0&0&0&0&0&0&x^3\\
0&0&0&0&0&0&y^3&0\\
0&0&0&0&0&z^3&0&0\\
0&0&0&0&w^3&0&0&0\\
0&0&0&\bar{w}^3&0&0&0&0\\
0&0&\bar{z}^3&0&0&0&0&0\\
0&\bar{y}^3&0&0&0&0&0&0\\
\bar{x}^3&0&0&0&0&0&0&0
\end{pmatrix},\quad
X^4 = \begin{pmatrix}
0&0&0&0&0&0&x^4&0\\
0&0&0&0&0&0&0&y^4\\
0&0&0&0&z^4&0&0&0\\
0&0&0&0&0&w^4&0&0\\
0&0&\bar{z}^4&0&0&0&0&0\\
0&0&0&\bar{w}^4&0&0&0&0\\
\bar{x}^4&0&0&0&0&0&0&0\\
0&\bar{y}^4&0&0&0&0&0&0
\end{pmatrix}
$$

$$
X^5 = \begin{pmatrix}
0&0&0&0&0&x^5&0&0\\
0&0&0&0&y^5&0&0&0\\
0&0&0&0&0&0&0&z^5\\
0&0&0&0&0&0&w^5&0\\
0&\bar{y}^5&0&0&0&0&0&0\\
\bar{x}^5&0&0&0&0&0&0&0\\
0&0&0&\bar{w}^5&0&0&0&0\\
0&0&\bar{z}^5&0&0&0&0&0
\end{pmatrix},\quad
X^6 = \begin{pmatrix}
0&0&0&0&x^6&0&0&0\\
0&0&0&0&0&y^6&0&0\\
0&0&0&0&0&0&z^6&0\\
0&0&0&0&0&0&0&w^6\\
\bar{x}^6&0&0&0&0&0&0&0\\
0&\bar{y}^6&0&0&0&0&0&0\\
0&0&\bar{z}^6&0&0&0&0&0\\
0&0&0&\bar{w}^6&0&0&0&0
\end{pmatrix}
$$

$$
X^7 = \begin{pmatrix}
0&0&0&x^7&0&0&0&0\\
0&0&y^7&0&0&0&0&0\\
0&\bar{y}^7&0&0&0&0&0&0\\
\bar{x}^7&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&z^7\\
0&0&0&0&0&0&w^7&0\\
0&0&0&0&0&\bar{w}^7&0&0\\
0&0&0&0&\bar{z}^7&0&0&0
\end{pmatrix},\quad
X^8 = \begin{pmatrix}
0&0&x^8&0&0&0&0&0\\
0&0&0&y^8&0&0&0&0\\
\bar{x}^8&0&0&0&0&0&0&0\\
0&\bar{y}^8&0&0&0&0&0&0\\
0&0&0&0&0&0&z^8&0\\
0&0&0&0&0&0&0&w^8\\
0&0&0&0&\bar{z}^8&0&0&0\\
0&0&0&0&0&\bar{w}^8&0&0
\end{pmatrix}
$$

$$
X^9 = \begin{pmatrix}
0&x^9&0&0&0&0&0&0\\
\bar{x}^9&0&0&0&0&0&0&0\\
0&0&0&y^9&0&0&0&0\\
0&0&\bar{y}^9&0&0&0&0&0\\
0&0&0&0&0&z^9&0&0\\
0&0&0&0&\bar{z}^9&0&0&0\\
0&0&0&0&0&0&0&w^9\\
0&0&0&0&0&0&\bar{w}^9&0
\end{pmatrix}.
\tag{4.5}
$$

It is possible to write the action on the probe brane in manifest $\mathcal{N} = (1,1)$ language by using the *three-dimensional* $\mathcal{N} = 1$ superfield notation described in [56]. By taking the superfields to be independent on the last space coordinate we get the $\mathcal{N} = (1,1)$ superfields. We will use the same three dimensional notation of [56] and define a vector superfield $\mathbf{\Gamma}_\alpha$, containing the gauge potential and the field X^2, coming from the third component of the three dimensional gauge fields. We also introduce the associated superfield strength

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$\mathbf{W}_\alpha = (1/2)D^\beta D_\alpha \mathbf{\Gamma}_\beta$ and supercovariant derivative[4] $\nabla_\alpha = D_\alpha + i[\mathbf{\Gamma}_\alpha, \cdot]$. The 7 scalar superfields $\mathbf{X}^i$ parameterizing the motion in the orbifold directions have projections: $\mathbf{X}^i| = X^i$, $\nabla_\alpha \mathbf{X}^i| = \psi^i_\alpha$ and $\nabla^2 \mathbf{X}^i| = F^i$, where F^i is the auxiliary field in $\mathbf{X}^i$.

The $\mathcal{N} = (1,1)$ Yang-Mills theory (sigma model) can then be written as

$$S = \frac{1}{g^2} \int d\sigma^2 \, d\theta^2 \mathrm{tr} \left(\mathbf{W}^\alpha \mathbf{W}_\alpha - \frac{1}{2} \nabla^\alpha \mathbf{X}^i \nabla_\alpha \mathbf{X}^i - \frac{i}{3} \omega^{ijk} \mathbf{X}^i \mathbf{X}^j \mathbf{X}^k \right), \tag{4.6}$$

where ω^{ijk} is the invariant 3rd-rank antisymmetric tensor of G_2, also known as the structure constant of the imaginary octonions. In our choice of indices for the coordinates, running from 3 to 9, the non zero elements of ω are

$$\omega^{789} = \omega^{569} = \omega^{468} = \omega^{394} = \omega^{358} = \omega^{367} = \omega^{475} = 1 \,, \tag{4.7}$$

the other following by the total antisymmetry of ω.

The very non trivial fact that makes the superpotential in (4.6) work is that squaring the 7 F-terms

$$F^i = i\omega^{ijk} X^j X^k \,, \tag{4.8}$$

one reproduces the usual quartic bosonic potential:

$$\sum_i \mathrm{tr} \left(F^i F^i \right) = - \sum_{i<j} \mathrm{tr} \left([X^i, X^j]^2 \right). \tag{4.9}$$

Thus, the vanishing of the 7 F-terms is enough to guarantee the vanishing of all 21 relative commutators.

The 7-vector F^i (4.8) is the analogue to our case of the moment map of [47, 48]! (We expand more on this at the end of next section). We will refer to it as the "octonionic" moment map because of the presence of the structure constant for the octonions.

It is amusing to notice that the same expression (4.6) seems to hold for the maximally supersymmetric case, where we drop the restrictions coming from the orbifold projection and consider the superfields to be arbitrary hermitian matrices. Thus, for the three dimensional $\mathcal{N} = 8$ theory, (which has exactly the same form as (4.6), only integrated over $d\sigma^3$), we have a manifestly $\mathcal{N} = 1$ formulation in which the manifest R-symmetry is G_2.

5. Probe analysis and the removal of the singularity at the origin

As usual, there are two possible branches that can be studied. The one of interest here is the "Higgs branch", defined by setting the field X^2 equal to a constant times the identity matrix. The other branch, known as the "Coulomb branch" is relevant, for instance, in the study of fractional branes.

The Higgs branch of the vacuum solution for the singular orbifold is given by the vanishing of the octonionic moment map,

$$F^i = 0 \,, \tag{5.1}$$

which has solutions iff all the X^i commute with each other.

[4] Although our gauge theory is abelian, the couplings (charges) of the matter fields to the gauge fields come from reducing the maximally supersymmetric theory and thus can be conveniently expressed as commutators by putting together the eight U(1) gauge fields into a diagonal matrix.

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It is not difficult to see that the vacuum equations imply $|x^i| = |y^i| = |z^i| = |w^i| := r^i$, so that we can set $x^i = r^i e^{i\chi^i}$, $y^i = r^i e^{i\psi^i}$, $z^i = r^i e^{i\zeta^i}$, $w^i = r^i e^{i\omega^i}$. Plugging these expressions back to the vacuum condition, we get 42 real equations (two from each commutator) for the 28 phases (there are 4 phases for each matrix X^i). It would naively seem that the system is over-determined, but 21 of the equations are redundant and we can solve in terms of 7 appropriately chosen phases. It is straightforward to check that a set of such phases is given by $\{\chi^3, \psi^4, \zeta^5, \omega^6, \zeta^7, \omega^8, \omega^9\}$. The rest of the phases are then expressed as linear combinations of the above: $\psi^3 = -\omega^9 + \psi^4$, $\zeta^3 = -\omega^8 + \zeta^5, \ldots$ etc. On the other hand, under a gauge transformation parameterized by $\{\Lambda_1, \ldots \Lambda_8\}$, $\{\chi^3, \psi^4, \zeta^5, \omega^6, \zeta^7, \omega^8, \omega^9\}$ get shifted by $\{\Lambda_1 - \Lambda_8, \ldots, \Lambda_7 - \Lambda_8\}$ respectively. A careful analysis reveals that *all* the phases can be gauge-fixed to zero modulo π. Consequently we are left with a 7 dimensional moduli space parameterized by $\pm r^i$. This is precisely the orbifold $\mathbb{R}^7/\Gamma$ we started with and it is, of course, singular at the origin. It is also straightforward to see, after integrating out the gauge field, that the metric is flat.

We are familiar with the fact that the coupling of the twisted metric moduli to the brane modifies the bosonic potential in such a way that the supersymmetric vacuum is now smoothed out. This usually comes about via D-term couplings, such as those generated by Fayet-Iliopoulos terms. Here we face an immediate puzzle because in the model we are considering there are neither D-terms nor twisted metric moduli. In fact, looking at the expansion of $\mathbf{\Gamma}_\alpha$ we see that the only scalar field is $B = D^\alpha \mathbf{\Gamma}_\alpha|$ which can be completely gauged away. In the reduction to two dimensions, we have the possibility of constructing a superfield $\mathbf{\Xi} = \mathbf{D}_\alpha \gamma^{\mathbf{2}}_{\alpha\beta} \mathbf{\Gamma}_\beta$, which is gauge invariant ($\gamma^2_{\alpha\beta}$ is the last of the Dirac matrices in three dimensions that becomes the chirality matrix in two), but that does not help either because its last component $F_{01} = D^2 \mathbf{\Xi}|$ is the electro-magnetic field strength and not an auxiliary field.

If there is a term that removes the singularity, it must come from a *direct* coupling of the twisted fields ϕ_h ($h \in \{\alpha, \beta, \gamma, \alpha\beta, \alpha\gamma, \beta\gamma, \alpha\beta\gamma\}$) to the matter superfields $\mathbf{X}^i$. There is one "almost" obvious candidate,[5] the coupling of each twisted field ϕ_h to a "twisted" mass term $\mathrm{tr}(\mathbf{X}^i \mathbf{X}^i \rho(h))$. The part of this coupling that is relevant is

$$S' = \frac{1}{g^2} \int d\sigma^2 \, d\theta^2 \sum_{h \neq e} \phi_h \mathrm{tr} \left(\mathbf{X}^i \mathbf{X}^i \rho(h) \right) . \tag{5.2}$$

In the presence of this coupling, equation (5.1) for the Higgs branch is now given by

$$F^i := i\omega^{ijk} X^j X^k - \sum_{h \neq e} \phi_h \left\{ X^i, \rho(h) \right\} = 0 \tag{5.3}$$

F^i being the modified (compared to (4.8)) "octonionic" moment map.

We now show that this modification removes the singularity at the origin. First of all, notice that, contrary to what happens with the usual D-terms, here the origin is always a solution to (5.3). However, we shall see that for generic values of ϕ_h the origin is an isolated point.

[5] One way to check this would be to perform an explicit string calculation of the couplings $\langle \phi\phi XX \rangle$ and $\langle \phi XXX \rangle$.

The first thing to notice about (5.3) is that each choice of i gives a set of four complex equations and their complex conjugates, equivalently, eight real equations. What does the trick is that, because of the particular structure of the non zero elements of ω^{ijk} in (4.7), the commutator $[X^j, X^k]$ has non-zero entries at the same places as X^i does (cf. (4.5)). The form of ω^{ijk} is of course dictated by invariance under the orbifold group Γ.

A second important property is that for each $\rho(h)$ there are four X's that anti-commute with it and three that commute. In particular, the coupling in (5.2) is non zero only between ϕ_h and those superfields that are *not* twisted by $R(h)$. However, one can easily see that it is possible to choose ϕ_h in such a way that each of the $7 \times 4 = 28$ complex fields making up the X's will appear linearly in one of the (5.3). (In fact three non zero ϕ's are enough).

Assuming that we have done as above and turned on a "mass" term for each of the 28 fields we see that the origin must be an isolated point. In fact, near the origin, the quadratic piece can be neglected and the equations simply impose $X^i \equiv 0$. What is more difficult is to show that there is a smooth solution away from the origin. This can be done to first order in perturbation theory. Let us collectively denote by t^A, $A = 1, \ldots, 56$ the real variables appearing in (5.3). For instance, one could set $x^3 = t^1 + it^2$, $y^3 = t^3 + it^4, \ldots, w^9 = t^{55} + it^{56}$. The superpotential becomes

$$W[t] = \frac{1}{6} \sum_{ABC} Q^{ABC} t^A t^B t^C + \frac{1}{2} \sum_A m^A t^A t^A \,, \tag{5.4}$$

where the totally symmetric tensor Q is determined by ω and the "mass" terms m^A are just linear combinations of ϕ's. Equations (5.3) can then be rewritten as a set of 56 real equations

$$\frac{1}{2} \sum_{BC} Q^{ABC} t^B t^C = m^A t^A \,, \tag{5.5}$$

Let us write $t^A = \hat{t}^A + \tau^A$ where $\hat{t}^A$ is a solution of the undeformed equations

$$\frac{1}{2} \sum_{BC} Q^{ABC} \hat{t}^B \hat{t}^C = 0 \,, \tag{5.6}$$

and τ^A is a perturbation. Moreover, let us assume that all $|m^A/\hat{t}^A|$, $|\tau^A/\hat{t}^A| \ll 1$. Expanding to first order we obtain an inhomogeneous linear equation for the τ's

$$\sum_{BC} Q^{ABC} \hat{t}^B \tau^C = m^A \hat{t}^A \,. \tag{5.7}$$

We require that the system have a 14 dimensional space of solutions, corresponding to the 7 coordinates of the manifold and 7 gauge directions. We have performed the computation using Mathematica and found that it does! (Not surprisingly, one can check that the addition of an eighth "untwisted" mass term $\sim \operatorname{tr}\left(\mathbf{X}^i \mathbf{X}^i\right)$ to the potential would lift the Higgs branch completely).

The perturbation theory we used is good far from the origin whereas near the origin we have used another approximation to show that the origin is an isolated point. This is what can be shown from this rather general analysis. What has *not* been shown is that the new, perturbed, solution is smooth everywhere. Our arguments do not rule out the possibility that a new (different from the origin) singularity develops. The most direct way to settle this would be to obtain the exact solution to the perturbed equation. We hope to report on this in some future work.

Let us now compare to the hyperKähler quotient construction [49, 47, 48]. The ingredients are: a large parent space $\mathcal{M}$, a gauge group G, a Lie-algebra-valued triplet μ^i, $i = 1, 2, 3$ and a triplet of numbers ζ^i valued in the center $\mathcal{Z}$ of $G/\mathrm{U}(1)$. i.e.

$$\begin{aligned} \mu^i : \mathcal{M} &\to \mathbb{R}^3 \otimes \mathcal{L}(G/\mathrm{U}(1)) \\ \zeta^i &\in \mathbb{R}^3 \otimes \mathcal{Z}\,. \end{aligned} \tag{5.8}$$

The hyperKähler quotient is then simply $\mu^{-1}(\zeta^i)/(G/\mathrm{U}(1))$.

These objects fit into a nice physical picture [41]: $\mathcal{M}$ is the space parameterized by the hypermultiplets of the low-energy effective theory on the probe, G is the gauge group of the latter, μ are the D-terms of the theory and ζ^i are the Fayet-Iliopoulos terms. The hyperKähler quotient in this language is the moduli space of vacua of the theory on the probe, i.e. the set of gauge-invariant solutions to the D-flatness conditions.

In our case the analogue of the FI terms are the 7 real moduli ϕ_h and the analogue of the moment map is the, now ϕ_h-dependent, octonionic map of (5.3). We have,

$$\begin{aligned} \mathbf{F}(\phi) : \mathcal{M} &\to \mathbb{R}^7 \otimes \mathcal{L}(G/\mathrm{U}(1)) \\ \phi_h &\in \mathbb{R}^7 \otimes \mathcal{Z}\,. \end{aligned} \tag{5.9}$$

The modified vacuum moduli space is simply $\mathrm{Ker}(\mathbf{F}(\phi))/(G/\mathrm{U}(1))$.

6. Conclusions

We have seen that D-branes can be useful tools in the study of G_2 orbifolds and proposed that they remove orbifold singularities in a novel way — by a twisted mass term instead of the D-terms. In the process of doing so, we have come across a proposal for a generalization of the moment map.

Clearly, our construction generalizes to many other situations and there are various possible directions along these lines. An understanding of the possible discrete subgroups of G_2 would tell us which are the interesting cases to be studied. A better understanding of the geometry of these orbifolds is needed, perhaps even explicit metrics can be constructed this way.

Unfortunately we have not been able to prove that our proposed bulk coupling renders the vacuum moduli space smooth everywhere. The most direct way to show this would be to obtain the exact metric on the modified vacuum moduli space. What we have shown is that a) the singularity at the origin is removed (becomes an isolated point) and b) the first-order perturbation does not lift the space of vacua.

It would clearly be interesting to justify (or rule out!) the presence of the twisted mass term by a direct string world-sheet calculation.

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